

# The Phases of Non-Compact QED<sub>3</sub>

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Non-compact three-dimensional QED is studied by computer simulations to understand its chiral symmetry breaking features for different values of the number of fermion flavors  $N_f$ . We consider the four-component formulation for the fermion fields, which arises naturally as the continuum limit of the staggered fermion construction in  $(2+1)$  dimensions. We present preliminary results for the equation of state of the theory in an effort to understand the properties of the chiral phase transition of the theory at a critical number of fermion flavors  $N_{fc}$ . Our preliminary results indicate that  $N_{fc} \approx 1.5$ .

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## 1. Introduction

The study of QFTs in which the ground state shows a sensitivity to the number of fermion flavors  $N_f$  is intrinsically interesting. According to approximate solutions of continuum Schwinger-Dyson equations (SDEs), QED<sub>3</sub> displays this phenomenon. It is believed to be confining and exhibit features such as dynamical mass generation when the number of fermion flavors  $N_f$  is smaller than a critical value  $N_{fc}$  [1, 2, 3, 4, 5, 6, 7]. Apparently, for  $N_f > N_{fc}$ , the attractive interaction between a fermion and an antifermion due to photon exchange is overwhelmed by the fermion screening of the theory's electric charge. Furthermore, over the past few years QED<sub>3</sub> has attracted a lot of attention, because of potential applications to models of high  $T_c$  superconductivity [8, 9, 6, 10, 11]. It is also an interesting and challenging model field theory which is being seen as an ideal laboratory to study more complicated gauge field theories.

Initial studies based on SDEs using the photon propagator derived from the leading order  $1/N_f$  expansion, where  $N_f$  is the number of fermion flavors, suggested that for  $N_f$  less than  $N_{fc} \simeq 3.2$  chiral symmetry is broken [1]. The model in the limit  $N_f \rightarrow N_{fc}$  is supposed to undergo an infinite-order phase transition [2]. Other studies taking non-trivial vertex corrections into account predicted chiral symmetry breaking for arbitrary  $N_f$  [3]. Studies which treat the vertex consistently in both numerator and denominator of the SDEs have found  $N_{fc} < \infty$ , with a value either in agreement with the original study [4], or slightly higher  $N_{fc} \simeq 4.3$  [5]. An argument based on a thermodynamic inequality predicted  $N_{fc} \leq \frac{3}{2}$  [12], a result that was later on challenged in [11]. A gauge invariant determination of  $N_{fc}$  based on the divergence of the chiral susceptibility gives  $N_{fc} \approx 2.16$  [6]. Recent progress in the direction of gauge covariant solutions for the propagators of QED<sub>3</sub> showed that in the Landau gauge a chiral phase transition exists at  $N_{fc} \approx 4$  [7]. Furthermore, a perturbative analysis of RG flow equations in the large- $N_f$  limit predicts  $N_{fc} \approx 6$  [13].

There have also been numerical attempts to resolve the issue via lattice simulations of QED<sub>3</sub>. Once again, opinions have divided on whether  $N_{fc}$  is finite [14, 15], or whether chiral symmetry is broken for all  $N_f$  [16]. A numerical study of the quenched ( $N_f = 0$ ) case has shown that chiral symmetry is broken [17]. More recent numerical results showed that chiral symmetry is also broken for  $N_f = 1$ , whereas  $N_f = 2$  appeared chirally symmetric with an upper bound of  $10^{-4}$  on the dimensionless condensate [18]. The principal obstruction to a definitive answer has been large finite volume effects resulting from the presence of a massless photon in the spectrum, which prevent a reliable extrapolation to the thermodynamic limit. Recent lattice simulations of the three-dimensional Thirring model, which may have the same universal properties as QED<sub>3</sub>, predicted  $N_{fc} = 6.6(1)$  [19]. In this paper we present a study of the QED<sub>3</sub> Equation of State based on preliminary results extracted from lattice simulations on large lattices (up to  $80^3$ ).

## 2. The Model

We are considering the four-component formulation of QED<sub>3</sub> where the Dirac algebra is represented by the  $4 \times 4$  matrices  $\gamma_0$ ,  $\gamma_1$  and  $\gamma_2$ . This formulation preserves parity and gives each spinor a global  $U(2)$  symmetry generated by  $\mathbf{1}$ ,  $\gamma_3$ ,  $\gamma_5$  and  $i\gamma_3\gamma_5$ ; the full symmetry is then  $U(2N_f)$ . If the fermions acquire dynamical mass the  $U(2N_f)$  symmetry is broken spontaneously to  $U(N_f) \times U(N_f)$  and  $2N_f^2$  Goldstone bosons appear in the particle spectrum.

The action of the lattice model we study is

$$S = \frac{\beta}{2} \sum_{x, \mu < \nu} \Theta_{\mu\nu}(x) \Theta_{\mu\nu}(x) + \sum_{i=1}^N \sum_{x, x'} \tilde{\chi}_i(x) M(x, x') \chi_i(x') \quad (2.1)$$

$$\Theta_{\mu\nu}(x) \equiv \theta_{x\mu} + \theta_{x+\hat{\mu}, \nu} - \theta_{x+\hat{\nu}, \mu} - \theta_{x\nu}$$

$$M(x, x') \equiv m\delta_{x, x'} + \frac{1}{2} \sum_{\mu} \eta_{\mu}(x) [\delta_{x', x+\hat{\mu}} U_{x\mu} - \delta_{x', x-\hat{\mu}} U_{x-\hat{\mu}, \mu}^{\dagger}].$$

This describes interactions between  $N$  flavors of Grassmann-valued staggered fermion fields  $\chi, \tilde{\chi}$  defined on the sites  $x$  of a three-dimensional cubic lattice, and real photon fields  $\theta_{x\mu}$  defined on the link between nearest neighbour sites  $x, x + \hat{\mu}$ . Since  $\Theta^2$  is unbounded from above, eq.(2.1) defines a non-compact formulation of QED; note however that to ensure local gauge invariance the fermion-photon interaction is encoded via the compact connection  $U_{x\mu} \equiv \exp(i\theta_{x\mu})$ , with  $U_{x+\hat{\mu}, -\mu} = U_{x\mu}^*$ . In the fermion kinetic matrix  $M$  the Kawamoto-Smit phases

$$\eta_{\mu}(x) = (-1)^{x_1 + \dots + x_{\mu-1}} \quad (2.2)$$

are designed to ensure relativistic covariance in the continuum limit, and  $m$  is the bare fermion mass.

If the physical lattice spacing is denoted  $a$ , then in the continuum limit  $a\partial \rightarrow 0$ , eq.(2.1) can be shown to be equivalent up to terms of  $O(a^2)$  to

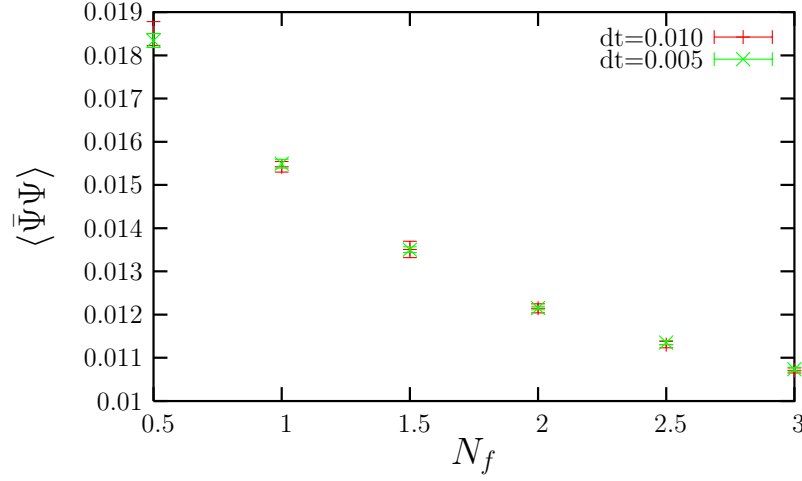
$$S = \sum_{j=1}^{N_f} \bar{\psi}^j [\gamma_{\mu} (\partial_{\mu} + igA_{\mu}) + m] \psi^j + \frac{1}{4} F_{\mu\nu} F_{\mu\nu} \quad (2.3)$$

ie. to continuum QED in 2+1 euclidean dimensions, with  $\psi, \bar{\psi}$  describing  $N_f$  flavors of four-component Dirac spinor acted on by  $4 \times 4$  matrices  $\gamma_{\mu}$ , and  $N_f \equiv 2N$ . The continuum photon field is related to the lattice field via  $\theta_{x\mu} = agA_{\mu}(x)$ , with dimensional coupling strength  $g$  given by  $g^2 = (a\beta)^{-1}$ , and the field strength  $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ . The continuum limit is thus taken when the dimensionless inverse coupling  $\beta \rightarrow \infty$ .

As reviewed in [18], for  $a > 0$  in the chiral limit the lattice action (2.1) retains only a remnant of the  $U(2N_f)$  global symmetry of (2.3) under global chiral/flavor rotations, namely a  $U(N) \otimes U(N)$  symmetry which is broken to  $U(N)$  either explicitly by the bare mass  $m \neq 0$ , or spontaneously by a chiral condensate  $\langle \tilde{\chi}\chi \rangle \neq 0$ , in which case the spectrum contains  $N^2$  exact Goldstone modes. It is expected that the symmetry breaking pattern  $U(2N_f) \rightarrow U(N_f) \otimes U(N_f)$  is restored in the continuum limit, implying the existence of an additional  $7N^2$  approximate Goldstone modes whose masses vanish as  $\beta \rightarrow \infty$ .

### 3. Numerical Simulations

In this section we present results from numerical simulations performed using the standard Hybrid Molecular Dynamics (HMD) R-algorithm. In order to ensure that the  $O(N^2 dt^2)$  systematic errors ( $dt$  is the time step of the HMD trajectory) are negligible, we performed two sets of simulations one with  $dt = 0.010$  and another with  $dt = 0.005$  on  $80^3$  lattices with  $m = 0.005$  and



**Figure 1:** Chiral condensate versus  $N_f$  from simulations with  $dt = 0.010$  and  $dt = 0.005$ ;  $\beta = 0.90$ ,  $m = 0.005$ , and  $L = 80$ .

$\beta = 0.90$ . As shown in Fig. 1 the values of the chiral condensate from the two sets of simulations with  $N_f = 0.5, \dots, 3.0$  agree within statistical error, implying that the systematic effects are negligible.

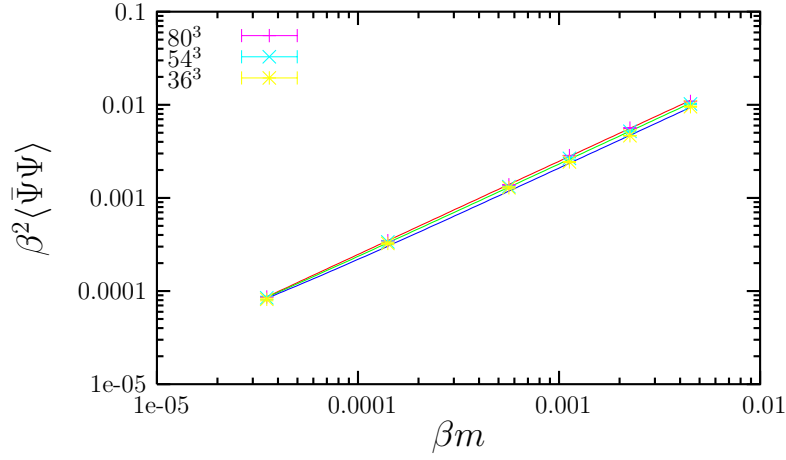
Next we present results from simulations with  $N_f = 1.5$  and  $N_f = 2.0$  (Figs. 2 and 3). More specifically, we study the behavior of the dimensionless chiral condensate  $\beta^2 \langle \bar{\psi}\psi \rangle$  versus the dimensionless fermion bare mass  $\beta m$  on different lattice sizes. In the  $N_f = 1.5$  case the finite size discrepancy between the  $L = 54$  and  $L = 80$  results is  $\sim 7\%$ , implying that the finite size effects for  $L = 80$  are small. In the  $N_f = 2.0$  case the discrepancy is  $\sim 1\%$ , implying that in this case the values of the condensate extracted from the  $80^3$  simulations should be even closer to the thermodynamic limit values.

For  $N_f = 2$ , we fitted the  $54^3$  data to

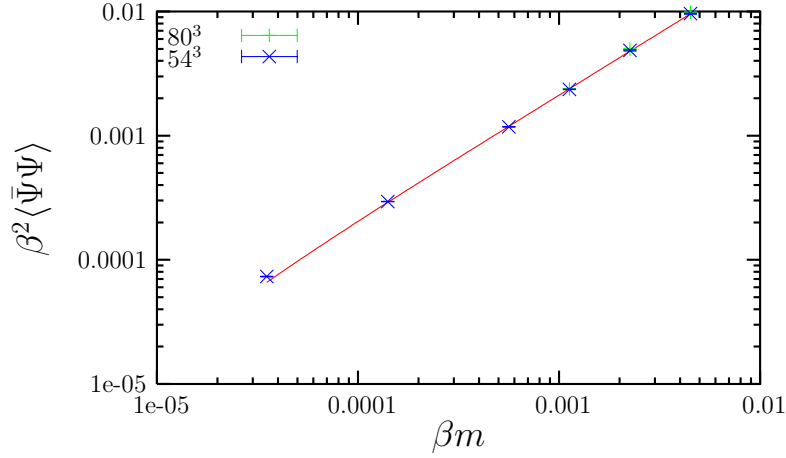
$$\beta^2 \langle \bar{\psi}\psi \rangle = a_0 + a_1 \cdot (\beta m) + a_2 \cdot (\beta m)^2. \quad (3.1)$$

The extracted value of  $a_0 = -8 \times 10^{-8}$  with a statistical error  $5 \times 10^{-7}$  and fit quality  $\chi^2/DOF = 0.9$  is consistent with zero with relatively high accuracy. For  $N_f = 1.5$ , we fitted the  $80^3$  data to (3.1) and got  $a_0 = -2 \times 10^{-6}$  with a statistical error  $10^{-6}$  and fit quality  $\chi^2/DOF = 4.1$ . This result is also very close to zero although the fit quality is lower than in the  $N_f = 2.0$  case. As we will see below the low fit quality could be attributed to the fact that  $N_f = 1.5$  may be close to the chiral phase transition.

We also checked the effects of lattice discretization on the values of the chiral condensate by comparing data extracted from simulations with  $\beta = 0.90$  and  $\beta = 1.20$  for  $N_f = 0.5, \dots, 2.0$ . We achieved this by fixing for the two sets of simulations the physical volume  $(L/\beta)^3$  and the physical mass  $\beta m$ . The results presented in Fig.4 show that the lattice discretization effects at  $\beta = 0.90$  are small for  $N_f > 0.5$ , whereas for  $N_f = 0.5$  there is an  $\sim 8\%$  discrepancy between the values of the condensate at  $\beta = 0.90$  and  $\beta = 1.20$ .



**Figure 2:** Dimensionless condensate versus dimensionless bare mass for  $N_f = 1.5$ .



**Figure 3:** Dimensionless condensate versus dimensionless bare mass for  $N_f = 2.0$ .

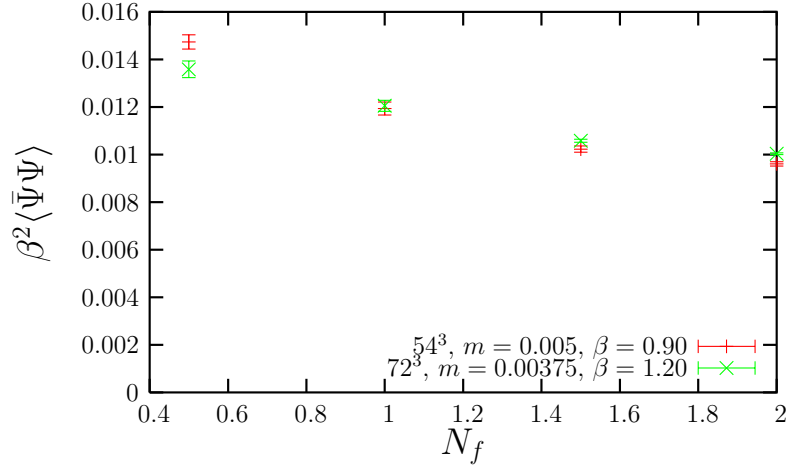
Next, we fitted the data for the dimensionless condensate at different values of  $N_f$  and  $m$ , and lattice sizes  $54^3$  and  $80^3$  to a renormalization group inspired equation of state that includes a finite size scaling term [20]:

$$m = A((\beta - \beta_c) + CL^{-\frac{1}{\nu}})(\beta^2 \langle \bar{\Psi} \Psi \rangle)^p + B(\beta^2 \langle \bar{\Psi} \Psi \rangle)^\delta, \quad (3.2)$$

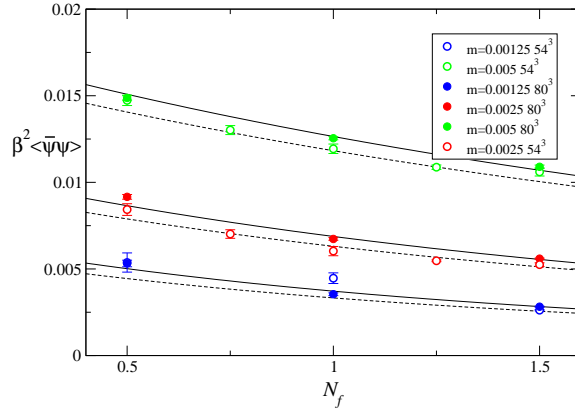
where  $p = \delta - 1/\beta_m$ . The results extracted from this fit are:  $A = 0.0477(38)$ ,  $B = 0.79(2)$ ,  $C = 10.7(8)$ ,  $N_{fc} = 1.52(6)$ ,  $\delta = 1.177(7)$ ,  $p = 0.73(2)$ . The data and the fitting functions are shown in Fig. 5. These results are consistent with a relatively smooth second order phase transition.

#### 4. Summary

We studied numerically the equation of state of non-compact QED<sub>3</sub>. The extrapolations to the chiral limit on lattices with small finite size effects show with high accuracy that the  $N_f = 2.0$  theory



**Figure 4:** Dimensionless condensate versus  $N_f$  at  $\beta = 0.90$  and  $1.20$  with constant  $L/\beta$  and  $\beta m$ .



**Figure 5:** Fits of  $\beta^2\langle\bar{\Psi}\Psi\rangle$  vs.  $N_f$  to a finite volume scaling form of the Equation of State.

is chirally symmetric with an accuracy of  $O(10^{-7})$ . The preliminary results extracted from fits to a finite volume Equation of State are consistent with a second order phase transition scenario and  $N_{fc} \approx 1.5$ . However, in order to reach a decisive conclusion on the value of  $N_{fc}$  and the properties of the chiral phase transition we have to extend our simulations to larger lattices, generate better statistics, and include more data points close to the continuum limit.

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